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Decay instability of an electron plasma wave in a dusty plasma

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The parametric decay instability of an electron plasma wave in a homogeneous, unmagnetized, hot and collisionless dusty plasma has been investigated analytically. The Vlasov equation has been solved perturbatively to find the nonlinear response of the plasma particles. The presence of the charged dust grains introduces a background inhomogeneous electric field that significantly influences the dispersive properties of the plasma and the decay process. The growth rate of the decay instability through the usual ion-acoustic mode is modified, and depends upon the dust perturbation parameter μ_i, dust correlation length q_0 and the related ion motion. However, the decay process of the electron plasma wave through the ultralow frequency dust mode, excited due to the presence of the dust particles, is more efficient than the decay through the usual ion-acoustic mode in the dusty plasma.

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I. INTRODUCTION

There has been a great interest in the generation of electrostatic electron plasma waves by the beating of two intense electromagnetic waves or, due to some other mode-coupling interactions in plasmas in recent years [1–13]. The most important applications of these electron plasma waves include laser-plasma beat wave accelerators and wake field accelerators [1–4], beat wave plasma heating and current drive [6,7], and space plasmas [8–13]. Besides the occurrence in the usual plasmas, electron plasma waves may be present in all dusty plasma environments due to mode-coupling interactions, beginning from laboratory devices to astrophysical and space plasmas.

Dusty plasmas having micrometer and submicrometer size; highly charged ($Z_g \approx 10^3$–10^4) and massive ($m_g/m_p \approx 10^6$–10^{18}) grains are gaining importance because of their relevance in the studies of space environments, asteroid zones, planetary rings, cometary tails, protostars, supernovae remanents as well as in the earth's environment [8–13]. On account of the presence of these heavily charged and massive dust particles, the interactions between dusty plasmas and external electric and magnetic fields may be considerably modified from those in the usual plasmas.

On account of the large amplitude nature of the electron plasma waves excited by either the beating of two electromagnetic waves or from some other mode-coupling interactions, viz., stimulated Raman scattering, the various nonlinear effects may come into play and become significant in a hot dusty plasma. Thus, the excited electron plasma waves may suffer a number of microinstabilities. Hence it is important to see the effects of highly charged and massive dust grains on the nonlinear interactions of these electron plasma waves in the dusty plasma. Simplest among all these nonlinear effects, decay instability [14] may play a vital role.

In the present paper, we have studied the nonlinear interaction of an electron plasma wave with a low frequency electrostatic plasma mode [15] (for example, ion-acoustic or some other low frequency mode due to the presence of dust grains), that is, the decay instability of the electron plasma wave into a low frequency electrostatic mode and another electron plasma wave in an unmagnetized dusty plasma. We consider a simple picture of the dusty plasma [8]: we assume that all the dust grains have the same size, mass, and are randomly oriented and that they are represented by point sources of electrostatic potential $\phi_0(\mathbf{x})$. The dusty plasma is assumed to be unmagnetized.

In Sec. II we solve the Vlasov equation to find the nonlinear response of electrons and ions in the dusty plasma perturbatively [8], where the static and massive charged dust grains are assumed to be randomly situated with a finite correlation having correlation length comparable to the wavelengths of the waves involved. In Sec. III, the nonlinear dispersion relation of the low frequency perturbation mode is obtained. The growth rate of the parametric decay instability of the electron plasma wave is presented in that section. In the same section, we have studied two important cases of the low frequency electrostatic mode for the decay instability of the electron plasma wave; one, the usual ion-acoustic mode, and the other, an ultralow frequency electrostatic mode due to the presence of the dust particles. Finally, a brief discussion of the results is given in Sec. IV.

II. KINETIC ANALYSIS FOR THE NONLINEAR RESPONSE OF ELECTRONS AND IONS IN A DUSTY PLASMA

We consider the propagation of a longitudinal electrostatic electron plasma wave (EPW) propagating in the z direction, in a homogeneous, unmagnetized, and colli-

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sionless dusty plasma with random distribution of highly charged ($Z_g = 10^3$–10^4) and massive (i.e., immobile) dust grains:

$$\mathbf{E}_0 = \hat{\mathbf{z}} E_0 \exp[-i(\omega_0 t - k_0 z)] \ , \tag{1}$$

$$\omega_0^2 = \omega_{pe}^2 + 3k_0^2 V_{\text{the}}^2/2 \ , \tag{2}$$

where ω_0 and $\mathbf{k}_0$ are the angular frequency and wave vector of the incident electrostatic wave. This electrostatic electron plasma wave may be a beat wave excited by two intense electromagnetic waves propagating in the plasma or excited by a nonlinear mode-coupling interaction, such as stimulated Raman scattering. The quantities $\omega_{pe} = (4\pi e^2 n_{0e}^0/m_e)^{1/2}$ and $V_{\text{the}} = (2T_e/m_e)^{1/2}$ are the electron plasma frequency and the electron thermal speed; $-e$, m_e, n_{0e}^0, T_e, and c being the electronic charge, mass, equilibrium number density of electrons, electron temperature in energy units, and the speed of light in a vacuum, respectively. The charged grains are considered to be so massive ($m_g \gg m_i$) that we consider the dynamics of electrons and ions in the background of the immobile grains [8]. The equilibrium thus consists of charged dust particles, ions, and electrons which satisfy the overall charge neutrality condition

$$\sum_\alpha q_\alpha n_{0\alpha}^0 + Q N_g = 0 \quad (\alpha = e, i) \ , \tag{3}$$

where $q_\alpha, n_{0\alpha}^0$, and Q, N_g are the charge and number density of electrons and ions and those of the dust grains, respectively. The stationary and randomly oriented dust particles give rise to a background electrostatic field which can be characterized by a potential function $\Phi_0(\mathbf{x})$. The average potential due to the dust grains is given by [8]

$$\bar{\Phi}_0 = \frac{1}{V}\int \phi_0(\mathbf{x}) d^3x = \frac{3Q}{\lambda_D}\left(\frac{\lambda_D}{r_0}\right)^3, \quad 4\pi r_0^3 N_g = 3 \tag{4}$$

where λ_D is the plasma Debye length and r_0 is the average distance between the grains. We make the assumption that the potential energy of a plasma particle in this field is much smaller than the thermal energy of electrons and ions: $\mu_\alpha = q_\alpha \bar{\Phi}_0/T_\alpha \ll 1$. This assumption is crucial to the perturbation technique employed in Ref. [8]. The quantity μ_α is known as the dust perturbation parameter.

Let us now consider the presence of a low frequency electrostatic mode $(\omega, \mathbf{k})$ in the dusty plasma. Due to the nonlinear interaction of the pump electron plasma wave $(\omega_0, \mathbf{k}_0)$ with the perturbation mode $(\omega, \mathbf{k})$, a sideband electrostatic electron plasma wave will be generated $(\omega_1, \mathbf{k}_1;\ \omega_1 = \omega - \omega_0,\ \mathbf{k}_1 = \mathbf{k} - \mathbf{k}_0)$. Both the generated sideband and the low frequency electrostatic mode will grow at the expense of the energy from the pump electron plasma wave, and this three-wave parametric instability is known as the parametric decay instability. The upper sideband $(\omega + \omega_0, \mathbf{k} + \mathbf{k}_0)$ is neglected as it is considered to be off resonant [14].

Since the dust grains have potential $\Phi_0(\mathbf{x})$ with a finite correlation having a correlation length comparable to the wavelengths of the waves involved, the motion of the plasma particles is described by the nonlinear Vlasov equation:

$$\frac{\partial f_\alpha^T}{\partial t} + (\mathbf{v}\cdot\nabla) f_\alpha^T + \frac{q_\alpha}{m_\alpha}\left(\mathbf{E}^T + \frac{1}{c}\mathbf{v}\times\mathbf{B}^T\right)\cdot\nabla_v f_\alpha^T = 0 \ , \tag{5}$$

where the superscript T denotes total quantity involved. We decompose f_α^T and $\mathbf{E}^T$ as

$$\begin{aligned} f_\alpha^T &= f_{0\alpha}^0 + f_{0\alpha} + f_{1\alpha} + f_\alpha \\ &= f_{0\alpha}^0 + f_{0\alpha}^L(\omega_0, \mathbf{k}_0) + f_{1\alpha}^L(\omega_1, \mathbf{k}_1) \\ &\quad + f_{1\alpha}^{NL}(\omega_1, \mathbf{k}_1) + f_\alpha^L(\omega, \mathbf{k}) + f_\alpha^D(\omega, \mathbf{k}) + f_\alpha^{NL}(\omega, \mathbf{k}) \ , \end{aligned} \tag{6}$$

$$\begin{aligned} \mathbf{E}^T &= \mathbf{E}_0 + \mathbf{E}_1 + \mathbf{E} \\ &= -\nabla\phi_0 - \nabla\phi_1 - \nabla\phi \ , \end{aligned} \tag{7}$$

where $f_{0\alpha}$, $f_{1\alpha}$, and f_α are the distribution functions of the dusty plasma corresponding to the pump wave, generated sideband, and the low frequency electrostatic mode, respectively. The superscripts L, D, and NL on the quantities denote, respectively, the linear, dust, and nonlinear terms. The quantities $\mathbf{E}_0$, $\mathbf{E}_1$, and $\mathbf{E}$ are the electric fields and ϕ_0, ϕ_1, and ϕ are the electrostatic potentials corresponding to the waves $(\omega_0, \mathbf{k}_0)$, $(\omega_1, \mathbf{k}_1)$, and $(\omega, \mathbf{k})$, respectively. We note here that B^T is zero for longitudinal electrostatic waves in an unmagnetized plasma. The unperturbed equilibrium distribution function $f_{0\alpha}^0$ in the stationary dusty plasma is given by [8]

$$\begin{aligned} f_{0\alpha}^0(\mathbf{x}, \mathbf{v}) = (1 - \mu_\alpha - \mu_\alpha^2)[&F_{0\alpha} + F_{0\alpha}' u_\alpha \sigma(\mathbf{x}) \\ &+ \tfrac{1}{2} F_{0\alpha}'' u_\alpha^2 \sigma^2(\mathbf{x})] \ , \end{aligned} \tag{8}$$

where the normalization factor is contained in $F_{0\alpha}$:

$$\begin{aligned} F_{0\alpha} &= (n_{0\alpha}^0/\pi\sqrt{\pi} V_{\text{th}\alpha}^3) \exp(-v^2/V_{\text{th}\alpha}^2) \ , \\ V_{\text{th}\alpha} &= (2T_\alpha/m_\alpha)^{1/2} \ . \end{aligned} \tag{9}$$

$u_\alpha = q_\alpha \bar{\Phi}_0$, T_α is the temperature of electrons and ions in energy units, and the prime on $F_{0\alpha}$ denotes derivative with respect to $\epsilon_\alpha = m_\alpha v^2/2$. In Eq. (8) the random statistical function $\sigma(\mathbf{x})$ defined by $q_\alpha \Phi_0(\mathbf{x}) = u_\alpha \sigma(\mathbf{x})$ is characterized by a correlation function

$$B(|\mathbf{x} - \mathbf{x}'|) = \langle \sigma(\mathbf{x}) \sigma(\mathbf{x}') \rangle \ , \tag{10}$$

where the bar over Φ_0 denotes ensemble average [16]. The spectral density of the correlation is given by

$$S(\mathbf{k}) = \frac{1}{(2\pi)^3}\int d^3x\, B(\mathbf{x}) \exp(i\mathbf{k}\cdot\mathbf{x}) \ , \tag{11}$$

so that

$$\langle \sigma(\mathbf{q}) \sigma(\mathbf{q}') \rangle = S(\mathbf{q}) \delta(\mathbf{q} + \mathbf{q}') \ . \tag{12}$$

Using Eq. (6) in Eq. (5) and carrying out the usual Laplace-Fourier transformation in the perturbation method [8], we obtain the linear distribution functions corresponding to the three waves as

$$\tilde{f}_{\alpha}^{L} = \frac{-iq_{\alpha}}{m_{\alpha}(\omega - \mathbf{k}\cdot\mathbf{v})} \int \mathbf{E}(\omega, \mathbf{k}_1)\cdot\nabla_v f_{0\alpha}^{0}(\mathbf{k}-\mathbf{k}_1) d^3k_1 \ , \tag{13}$$

$$\tilde{f}_{\alpha}^{D} = \frac{iq_{\alpha}^2}{m_{\alpha}^2(\omega - \mathbf{k}\cdot\mathbf{v})} \int d^3k_2 \Phi_0(\mathbf{k}_2)\mathbf{k}_2\cdot\nabla_v \int \frac{\mathbf{E}(\omega,\mathbf{k}_1)}{\omega - (\mathbf{k}-\mathbf{k}_2)\cdot\mathbf{v}} \cdot \nabla_v f_{0\alpha}^{0}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) d^3k_1 \ , \tag{14}$$

$$\tilde{f}_{0\alpha}^{L} = \frac{-iq_{\alpha}}{m_{\alpha}(\omega_0 - \mathbf{k}_0\cdot\mathbf{v})} \int \mathbf{E}_0(\omega_0, \mathbf{k}_1)\cdot\nabla_v f_{0\alpha}^{0}(\mathbf{k}_0-\mathbf{k}_1) d^3k_1 \ , \tag{15}$$

$$\tilde{f}_{1\alpha}^{L} = \frac{-iq_{\alpha}}{m_{\alpha}(\omega_1 - \mathbf{k}_1\cdot\mathbf{v})} \int \mathbf{E}_1(\omega_1, \mathbf{k}_1')\cdot\nabla_v f_{0\alpha}^{0}(\mathbf{k}_1-\mathbf{k}_1') d^3k_1' \ , \tag{16}$$

where a tilde over a quantity denotes Fourier-Laplace transformations. In Eqs. (15) and (16), we have neglected the effect of dust on the high frequency electron plasma waves; the low frequency waves are more likely to be affected by the dust particles [8,11].

Fourier-Laplace transforming Eq. (5) and then substituting Eqs. (15) and (16) in it, we obtain the nonlinear distribution function $\tilde{f}_{\alpha}^{NL}$ for the low frequency electrostatic mode due to the beating of the high frequency electron plasma wave (pump) and the generated sideband electron plasma wave $(\omega_1, \mathbf{k}_1)$:

$$\tilde{f}_{\alpha}^{NL} = \frac{-q_{\alpha}^2}{2m_{\alpha}^2(\omega - \mathbf{k}\cdot\mathbf{v})} \Bigg[\mathbf{E}_0\cdot\nabla_v \frac{1}{\omega_1 - \mathbf{k}_1\cdot\mathbf{v}} \int \mathbf{E}_1(\omega_1, \mathbf{k}_1')\cdot\nabla_v f_{0\alpha}^{0}(\mathbf{k}_1-\mathbf{k}_1') d^3k_1' + \mathbf{E}_1\cdot\nabla_v \frac{1}{\omega_0 - \mathbf{k}_0\cdot\mathbf{v}} \int \mathbf{E}_0(\omega_0, \mathbf{k}_1)\cdot\nabla_v f_{0\alpha}^{0}(\mathbf{k}_0-\mathbf{k}_1) d^3k_1 \Bigg] \ . \tag{17}$$

Similarly, Fourier-Laplace transforming Eq. (5) and then using the linear response, Eqs. (13)–(15) in it, we obtain the nonlinear distribution function $\tilde{f}_{1\alpha}^{NL}(\omega_1, \mathbf{k}_1)$ for the generated sideband electron plasma wave due to the beating of the pump electron plasma wave $(\omega_0, \mathbf{k}_0)$ and the low frequency electrostatic perturbation mode $(\omega, \mathbf{k})$:

$$\tilde{f}_{1\alpha}^{NL} = \frac{-q_{\alpha}^2}{2m_{\alpha}^2(\omega_1 - \mathbf{k}_1\cdot\mathbf{v})} \Bigg[\mathbf{E}\cdot\nabla_v \frac{1}{\omega_0 - \mathbf{k}_0\cdot\mathbf{v}} \int \mathbf{E}_0^*(\omega_0, \mathbf{k}_1)\cdot\nabla_v f_{0\alpha}^{0}(\mathbf{k}_0-\mathbf{k}_1) d^3k_1 + \mathbf{E}_0^*\cdot\nabla_v \frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}} \int \mathbf{E}(\omega, \mathbf{k}_1)\cdot\nabla_v f_{0\alpha}^{0}(\mathbf{k}-\mathbf{k}_1) d^3k_1 \Bigg] \ , \tag{18}$$

where the symbol $*$ denotes complex conjugate. In Eqs. (17) and (18), we have taken the dust contribution through the modified background distribution function only.

Now using Poisson's equation for the decay waves $(\omega, \mathbf{k})$ and $(\omega_1, \mathbf{k}_1)$, we have

$$k^2\phi = 4\pi \sum_{\alpha} q_{\alpha} \int d^3v (\tilde{f}_{\alpha}^{L} + \tilde{f}_{\alpha}^{D} + \tilde{f}_{\alpha}^{NL}) \ , \tag{19}$$

and

$$k_1^2\phi_1 = 4\pi \sum_{\alpha} q_{\alpha} \int d^3v (\tilde{f}_{1\alpha}^{L} + \tilde{f}_{1\alpha}^{NL}) \ . \tag{20}$$

From Eq. (8), it can be shown that

$$f_{0\alpha}^{0} = C_{\alpha}(\mathbf{k}) F_{0\alpha}(\mathbf{v}) \ , \tag{21}$$

where

$$C_{\alpha}(\mathbf{k}) = \delta(\mathbf{k}) - \mu_{\alpha}\sigma(\mathbf{k}) + \frac{\mu_{\alpha}^2}{2}\sigma^2(\mathbf{k}) \ . \tag{22}$$

In order to perform different integrations, we note that $\langle\sigma(\mathbf{k})\rangle = \langle\sigma^2(\mathbf{k})\rangle = \delta(\mathbf{k})$ and $\langle\sigma(\mathbf{k}_2)\sigma(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\rangle = S(\mathbf{k}_2)\delta(\mathbf{k}-\mathbf{k}_1)$ and, therefore, the ensemble average of $f_{0\alpha}^{0}(\mathbf{k}-\mathbf{k}_1)$ and $\phi(\mathbf{k}_2) f_{0\alpha}^{0}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)$ are obtained, respectively, as

$$\langle f_{0\alpha}^{0}(\mathbf{k}-\mathbf{k}_1)\rangle = \left(1 - \mu_{\alpha} + \frac{\mu_{\alpha}^2}{2}\right) F_{0\alpha}(\mathbf{v})\delta(\mathbf{k}-\mathbf{k}_1) \tag{23}$$

and

$$\langle \phi(\mathbf{k}_2) f_{0\alpha}^{0}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\rangle = \frac{u_{\alpha}}{q_{\alpha}} [\delta(\mathbf{k}_2)\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) - \mu_{\alpha} S(\mathbf{k}_2)\delta(\mathbf{k}-\mathbf{k}_1)] \ , \tag{24}$$

where we have retained terms up to the second order in the perturbation parameter μ_{α}. Further, we assume a model Gaussian distribution for the correlation function [8]

$$S(\mathbf{q}) = (1/\pi\sqrt{\pi}q_0^3)\exp(-q^2/q_0^2) , \tag{25}$$

where q_0 is the correlation length for the static dust grains in the plasma.

III. NONLINEAR DISPERSION RELATION AND GROWTH RATES

Case A: $kV_{\text{the}} > \omega > kV_{\text{th}i}$. We assume for simplicity that all the waves are propagating in the same direction ($\mathbf{k}_0\|\mathbf{k}_1\|\mathbf{k}\|\hat{\mathbf{z}}$). In the case when the frequency ω of the low frequency perturbation mode $(\omega,\mathbf{k})$ lies between kV_{the} and $kV_{\text{th}i}$, that is $kV_{\text{the}} > \omega > kV_{\text{th}i}$ for the ion-acoustic branch, after performing different integrations, Poisson's equation (19) finally takes the following form:

$$\epsilon\phi = C\phi_0\phi_1 , \tag{26}$$

where $\epsilon = 1+\chi+\chi_D$ is the dielectric function of the low frequency mode $(\omega,\mathbf{k})$; χ is the usual dielectric susceptibility of the low frequency mode and is given by [15]

$$\chi = \frac{1}{k^2\lambda_{Dc}^2} - \frac{\omega_{pi}^2}{\omega^2} + \frac{2i\sqrt{\pi}\omega_{pi}^2\omega}{k^3V_{\text{th}i}^3}\exp\left(-\frac{\omega^2}{k^2V_{\text{th}i}^2}\right) , \tag{27}$$

χ_D is the dielectric susceptibility of the low frequency mode $(\omega,\mathbf{k})$ due to the presence of the dust particles and we obtain it as

$$\chi_D = -\frac{\mu_e^2\omega_{pe}^2I_{De}^{(1)}}{\sqrt{\pi}k^2V_{\text{the}}^2} - \frac{\sqrt{\pi}\mu_i^2kq_0\omega_{pi}^2V_{\text{th}i}^2}{2\omega^4}\left(1+\frac{\sqrt{\pi}q_0}{2k}\right) - \frac{i\omega_e^2\omega_{pe}^2q_0I_{De}^{(2)}}{k^2\omega V_{\text{the}}} + \frac{i2\pi\mu_i^2\omega_{pi}^2I_{\text{D}i}^{(2)}}{q_0\omega V_{\text{th}i}} , \tag{28}$$

and the coupling coefficient C in Eq. (26) is obtained as

$$C \simeq \frac{e\omega_{pe}^2k_0k_1}{m_ek^3V_{\text{the}}^2}\left(\frac{k_0}{\omega_0^2}+\frac{k_1}{\omega_1^2}\right) . \tag{29}$$

In Eq. (28), the integrals $I_{De}^{(1)}$, $I_{De}^{(2)}$, and $I_{Di}^{(2)}$ are defined as

$$I_{De}^{(1)} = \int_0^\infty q_\| dq_\| \left[\frac{2}{q_0(k+q_\|)} - \frac{(k-q_\|)V_{\text{the}}^2}{\omega^2q_0}\right]\exp\left(-\frac{q_\|^2}{q_0^2}\right) , \tag{30}$$

$$I_{De}^{(2)} = \frac{1}{q_0^2}\int_0^\infty q_\| dq_\| \exp\left[-\frac{q_\|^2}{q_0^2} - \left(\frac{\omega}{(k-q_\|)V_{\text{the}}}\right)^2\right] , \tag{31}$$

and

$$I_{Di}^{(2)} = \int_0^\infty \frac{q_\| dq_\|}{(k+q_\|)^2}\exp\left[-\frac{q_\|^2}{q_0^2} - \left(\frac{\omega}{(k+q_\|)V_{\text{th}i}}\right)^2\right] . \tag{32}$$

In deriving Eq. (26), we have neglected the dust effect in the coupling coefficient C (which is due to the beating of the high frequency electrostatic waves) while retaining the effect of dust in the low frequency dielectric function ϵ through χ_D.

On evaluating the integrals involved in Eq. (20) we obtain the equation for the sideband $(\omega_1,\mathbf{k}_1)$ which is a high frequency electron plasma wave as

$$\epsilon_1\phi_1 = C_1\phi_0^*\phi , \tag{33}$$

where ϵ_1 is the usual dielectric function for the sideband high frequency electron plasma wave, and the coupling coefficient C_1 is obtained after simplification and retaining only the dominant terms:

$$C_1 \simeq -\frac{4ek_0\omega_{pe}^2\omega}{m_ek_1^2k\omega_1V_{\text{the}}^4} . \tag{34}$$

Eliminating ϕ from Eqs. (26) and (33), we obtain the following nonlinear dispersion relation for the low frequency perturbation mode $(\omega,\mathbf{k})$:

$$\epsilon = \frac{\mu}{\epsilon_1} , \tag{35}$$

where

$$\mu = |\phi_0|^2C_1C \tag{36}$$

is known as the coupling coefficient for the parametric decay instability and the coefficients C and C_1 are given by Eqs. (29) and (34). It may be noticed here that the coupling coefficient μ is real and positive as C is already positive and C_1 is also positive because $\omega_1 = \omega - \omega_0$ is negative.

The growth rate for the decay instability γ_0 is obtained from the following relation [14]:

$$\gamma_0^2 = -\frac{\mu}{(\partial\epsilon_r/\partial\omega)(\partial\epsilon_{1r}/\partial\omega_1)} , \tag{37}$$

where the subscript r represents the real part. After simplification and rearrangement of terms, we obtain the expressions for μ, $\partial\epsilon_r/\partial\omega$, and $\partial\epsilon_{1r}/\partial\omega_1$ as follows:

$$\mu \simeq -\frac{4|V_{0e}/V_{\text{the}}|^2\omega_{pe}^4\omega_0^2\omega}{k^4k_1\omega_1V_{\text{the}}^4}\left(\frac{k_0}{\omega_0^2}+\frac{k_1}{\omega_1^2}\right) , \tag{38}$$

$$\frac{\partial\epsilon_r}{\partial\omega} = \frac{2\omega_{pi}^2}{\omega^3}(1+f_D) , \tag{39}$$

and

$$\frac{\partial\epsilon_{1r}}{\partial\omega_1} = \frac{2\omega_{pe}^2}{\omega_1^3} , \tag{40}$$

where

$$|V_{0e}/V_{\rm the}| = \left|\frac{ek_0\phi_0}{m_e\omega_0 V_{\rm the}}\right| ,$$

$$f_D = \frac{\sqrt{\pi}\mu_i^2 k q_0 V_{\rm thi}^2}{\omega^2}\left(1+\frac{\sqrt{\pi}q_0}{2k}\right) \times \left(1-\beta\frac{1-\sqrt{\pi}q_0/2k}{1+\sqrt{\pi}q_0/2k}\right) , \quad (41)$$

$$\beta = \frac{1}{2\pi}\frac{\mu_e^2}{\mu_i^2}\frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{\omega^2}{k^2V_{\rm thi}^2} .$$

For calculating $\partial\epsilon_{1r}/\partial\omega_1$, we have taken the simplest expression for the dielectric function for the high frequency sideband electron plasma wave as

$$\epsilon_{1r} = 1 - \frac{\omega_{pe}^2}{\omega_1^2 - 3k_I^2 V_{\rm the}^2/2} . \quad (42)$$

The normalized growth rate γ_0/ω of the decay instability through the usual ion-acoustic mode, therefore, finally takes the following form:

$$\left(\frac{\gamma_0}{\omega}\right)_{\rm IA} = \left[\frac{|V_{0e}/V_{\rm the}|^2\omega_{pe}^2\omega_0^2\omega_1^2\omega^2}{k^4k_1\omega_{pi}^2V_{\rm the}^4(1+f_D)}\left(\frac{k_0}{\omega_0^2}+\frac{k_1}{\omega_1^2}\right)\right]^{1/2} . \quad (43)$$

The linear damping rate of the low frequency perturbation mode in the dusty plasma is obtained from [14]

$$\gamma_L = -\frac{\epsilon_i}{\partial\epsilon_r/\partial\omega} , \quad (44)$$

where the imaginary part of ϵ in the present case is given by

$$\epsilon_i = \frac{2\sqrt{\pi}\omega_{pi}^2\omega}{k^3V_{\rm thi}^3}\exp\left(-\frac{\omega^2}{k^2V_{\rm thi}^2}\right) + \frac{2\pi\mu_i^2\omega_{pi}^2 I_{Di}^{(2)}}{q_0\omega V_{\rm thi}} - \frac{\mu_e^2 q_0\omega_{pe}^2 I_{De}^{(2)}}{k^2\omega V_{\rm the}} . \quad (45)$$

The integrals $I_{De}^{(2)}$ and $I_{Di}^{(2)}$ are given by Eqs. (31) and (32), respectively.

In the presence of the linear damping of the low frequency ion-acoustic wave, Eq. (44), and neglecting the damping of the high frequency electrostatic decay wave $(\omega_1, \mathbf{k}_1)$, the overall growth rate of the decay instability is given by [14]

$$\gamma = [(\gamma_L^2 + 4\gamma_0^2)^{1/2} - \gamma_L]/2 . \quad (46)$$

From Eq. (43), we notice that the modification of the undamped growth rate of the decay instability of the electron plasma wave into an ion-acoustic wave and another electron plasma wave comes through the function f_D which contains the second order dust perturbation effect and the first order effect vanishes because of the $\mathbf{k}_2$ integration in Eq. (14). However, the overall growth rate γ is affected because of the damping of the low frequency ion-acoustic mode Eq. (44).

Case B: $\omega < kV_{\rm thi}$. We now consider the case of the extremely low frequency mode in the dusty plasma, $\omega < kV_{\rm thi}$. Following Ref. [15], one can obtain a new low frequency electrostatic mode in the dusty plasma having a frequency much less than the usual ion-acoustic branch. This ultralow frequency mode exists only due to the presence of the dust particles. The linear dielectric function of the dusty plasma in this case becomes [15]

$$\epsilon(\omega, \mathbf{k}) = \epsilon_r + i\epsilon_i , \quad (47)$$

where

$$\epsilon_r = 1 + \frac{1}{k^2\lambda_{De}^2}[1-2\mu_e+\mu_e^2(1-\tfrac{1}{3}{\rm Re}[Z(k/q_0)])] + \frac{1}{k^2\lambda_{Di}^2}(1-2\mu_i+\mu_i^2) - \frac{3\mu_i^2\omega_{pi}^2q_0^2}{2\omega^2k^2} , \quad (48)$$

$$\epsilon_i = -\frac{\mu_e^2}{3k^2\lambda_{De}^2}{\rm Im}[Z(k/q_0)] + \frac{\sqrt{\pi}\omega}{k^3\lambda_{Di}^2V_{\rm thi}}(1-2\mu_i+\mu_i^2)\exp\left(-\frac{\omega^2}{k^2V_{\rm thi}^2}\right) + \frac{\mu_i^2\omega_{pi}^2V_{\rm thi}}{k^2\omega^3q_0}\int_{-\inf}^{\infty} q_\parallel(k-q_\parallel)^2dq_\parallel \exp\left[-\frac{q_\parallel^2}{q_0^2}-\left(\frac{\omega}{(k-q_\parallel)V_{\rm thi}}\right)^2\right] - \frac{\mu_i^2\omega_{pi}^2\omega}{2k^2q_0V_{\rm thi}^3}\int_{-\infty}^{\infty}\frac{q_\parallel dq_\parallel}{(k-q_\parallel)^2}\exp\left[-\frac{q_\parallel^2}{q_0^2}-\left(\frac{\omega}{(k-q_\parallel)V_{\rm thi}}\right)^2\right] . \quad (49)$$

$Z(k/q_0)$ is the plasma dispersion function with argument k/q_0. Since $\mu_e, \mu_i < 1$, the dispersion relation of this ultralow frequency mode becomes [15]

$$\omega^2 = \frac{3\mu_i^2q_0^2\omega_{pi}^2}{2k^2(1+1/k^2\lambda_{De}^2+1/k^2\lambda_{Di}^2)} \simeq \tfrac{3}{4}\mu_i^2q_0^2V_{\rm thi}^2 . \quad (50)$$

This has no analog in the usual electron-ion plasma. This arises due to the oscillations of the ions in the static structure of the dust distribution.

Let us now consider the decay of the high frequency electron plasma wave into this ultralow frequency mode and another high frequency electron plasma wave. Following the same procedure of the previous case A, the undamped growth rate γ_0 of the decay instability can be obtained from Eq. (37), where now

$$\mu \simeq -\frac{4|V_{0e}/V_{the}|^2\omega_{pe}^4\omega_0^2\omega}{k^4k_1\omega_1V_{the}^4}\frac{n_{0i}^0T_e^2}{n_{0e}^0T_i^2}\left(\frac{k_0}{\omega_0^2}+\frac{k_1}{\omega_1^2}\right)\times\left(1-\frac{n_{0i}^0m_eT_e}{n_{0e}^0m_iT_i}\right), \tag{51}$$

$$\frac{\partial\epsilon_r}{\partial\omega}=\frac{3\mu_i^2\omega_{pi}^2q_0^2}{\omega^3k^2}, \tag{52}$$

and $\partial\epsilon_{1r}/\partial\omega_1$ is the same as in case A. Therefore, finally the normalized growth rate γ_0/ω of the decay instability through this ultralow frequency electrostatic new mode becomes

$$\left(\frac{\gamma_0}{\omega}\right)_{\rm ULF}=\left[\frac{2|V_{0e}/V_{the}|^2\omega_{pe}^2\omega_0^2\omega_1^2\omega^2}{3k_{k_1}^2q_0^2\omega_{pi}^2V_{the}^4\mu_i^2}\frac{n_{0i}^0T_e^2}{n_{0e}^0T_i^2}\left(\frac{k_0}{\omega_0^2}+\frac{k_1}{\omega_1^2}\right)\times\left(1-\frac{n_{0i}^0m_eT_e}{n_{0e}^0m_iT_i}\right)\right]^{1/2}. \tag{53}$$

The linear damping rate of this ultralow frequency perturbation mode in the dusty plasma is given by Eq. (44), where now the imaginary part of ϵ is given by Eq. (49). This is a different channel of decay in the dusty plasma. We can now compare the growth rate of decay instability through the usual ion-acoustic wave and the different ultralow frequency mode. From Eqs. (53) and (43), the ratio of the normalized growth rates through the ultralow frequency mode and the usual ion-acoustic mode is given by

$$\left(\frac{\gamma_0}{\omega}\right)_{\rm ULF}\Big/\left(\frac{\gamma_0}{\omega}\right)_{IA}\simeq\left[\frac{2}{3}\left(\frac{n_{0i}^0}{n_{0e}^0}\right)\left(\frac{T_e}{T_i}\right)^2\times\left(\frac{k}{q_0}\right)^2\left(\frac{1+f_D}{\mu_i^2}\right)\right]^{1/2}\gg 1\,. \tag{54}$$

Thus, we notice that the decay instability of the electron plasma wave through the ultralow frequency mode is more efficient than that through the usual ion-acoustic mode. The enhancement of the growth rate depends upon the non-neutrality of the electron and ion densities, temperature ratio, correlation length, and the dust perturbation parameter.

IV. DISCUSSION

We have investigated the nonlinear decay instability of an electron plasma wave in an unmagnetized and collisionless hot dusty plasma. The background inhomogeneous electric potential created by the highly charged and randomly oriented massive dust grains has significant effects on the low frequency electrostatic modes and the nonlinear decay process of the high frequency electron plasma waves. The normalized growth rate of the decay instability of the electrostatic electron plasma wave through the ion-acoustic mode is modified due to the presence of the dust. The modification of the decay instability depends upon the ion dust perturbation parameter μ_i, dust correlation length q_0, and the other quantities related to the motion of ions. However, the normalized growth rate of the decay instability through the ultralow frequency electrostatic mode due to the presence of the dust is much higher than that through the usual ion-acoustic branch of frequencies. Thus, the presence of highly charged and massive dust grains plays a vital role in the nonlinear mode-coupling interactions in a dusty plasma.

It may be mentioned here that various other nonlinear interactions of large amplitude electrostatic and electromagnetic waves with other possible modes in dusty plasmas are also of great importance, and work along these lines is in progress.

[1] B. I. Cohen, A. N. Kaufman, and K. M. Watson, Phys. Rev. Lett. **29**, 581 (1972); M. N. Rosenbluth and C. S. Liu, *ibid.* **29**, 701 (1972).

[2] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).

[3] J. L. Bobin, in *Proceedings of the ECFA-CAS/CERN-IN 2P3-IRF/CEA-EPS Workshop, Orsay, Italy, 1987*, edited by S. Turner (CERN, Geneva, 1987), Vol. I, p. 58.

[4] C. Joshi, C. E. Clayton, W. B. Mori, J. M. Dawson, and T. Katsouleas, Comments Plasma Phys. Controlled Fusion **16**, 65 (1994).

[5] M. Salimullah, Phys. Fluids **31**, 3785 (1988).

[6] B. I. Cohen, Comments Plasma Phys. Controlled Fusion **8**, 197 (1984); B. I. Cohen, R. H. Cohen, B. G. Logan, W. McCay Nevins, G. R. Smith, A. V. Kluge, and A. H. Kritz, Nucl. Fusion **28**, 1519 (1988).

[7] M. R. Amin and R. A. Cairns, Nucl. Fusion **30**, 327 (1990); Phys. Fluids B **3**, 151 (1991).

[8] V. N. Tsytovich, U. de Angelis, and R. Bingham, J. Plasma Phys. **42**, 445 (1989); U. de Angelis, A. Forlani, R. Bingham, P. K. Shukla, A. Ponomarev, and V. N. Tsytovich, Phys. Plasmas **1**, 136 (1994).

[9] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).

[10] P. K. Shukla, U. de Angelis, and L. Stenflo, Phys. Scr. **45**, 465 (1992).

[11] V. N. Tsytovich, G. E. Morfill, R. Bingham, and U. de Angelis, Comments Plasma Phys. Controlled Fusion **13**, 153 (1990); V. N. Tsytovich and O. Havnes, *ibid.* **15**, 267

(1993); V. N. Tsytovich, *ibid.* **15**, 349 (1994).
[12] T. G. Northrop, Phys. Scr. **45**, 475 (1992).
[13] R. C. Hazelton, IEEE Trans. Plasma Sci. **22**, 91 (1994).
[14] C. S. Liu and P. K. Kaw, in *Advances in Plasma Physics*, edited by A. Simon and W. B. Thompson (Wiley, New York, 1976), Vol. 6, p. 83; C. S. Liu and V. K. Tripathi, Phys. Rep. **130**, 144 (1986).
[15] M. Salimullah and A. Sen, Phys. Fluids B **4**, 1441 (1992); Phys. Lett. A **163**, 82 (1992).
[16] A. Ishimaru, *Wave Propagation and Scattering in Random Media* (Academic, New York, 1978), Vol. 2, Chap. 16.